You have been hired to design and decorate a house. Your goal is to come up with a design that can be implemented without going over budget.

## Your task is to calculate the costs associated with your design without going over the budget of $\$ 8000$. This includes the cost for the painters, the carpets, the curtains, the light fixtures and the new roofing shingles. You do not need to calculate the taxes.

## Part 1 - The Painters

You have already decided on the palette of colours that will be used throughout the house. You are considering two different companies to paint the rooms. Both companies charge for their work according to the following linear relations, where $\mathbf{C}(\mathbf{h})$ is the cost in dollars and $\mathbf{h}$ the number of hours:

Company A: $\quad \mathbf{C}(\mathrm{h})=24.5 \mathrm{~h}+130$
Company B: $\quad \mathbf{C}(\mathbf{h})=26.5 \mathrm{~h}+96$
You realise that for a certain number of hours both companies charge the same amount. Find that number of hours and use the cost associated with it in your budget.

## Part 2 - Carpets

You need to purchase carpets to cover the entire floor in the two bedrooms. The carpet you are considering is sold at $\$ 55$ per square metre. This price includes all the materials and the installation. To figure out the area that needs to be covered you are provided with a plan of the two bedrooms by the architect. He tells you that the areas of the two bedrooms are equal to one another. All dimensions are in meters.

Bedroom 1


Bedroom 2


## Part 3 - Curtains

You must purchase curtains for 12 windows. The store you are dealing with are offering you a deal based on the number of windows that need curtains.

The deal for the curtains follows a linear function where the cost depends on the number of windows as shown in the table.

| Cost for curtains |  |
| :---: | :---: |
| Number of windows <br> $(\mathrm{n})$ | Cost (\$) <br> C(n) |
| 5 | 877.50 |
| 9 | 1579.50 |

## Part 4 - Recessed Lighting

Because the light shines in the shape of a cone when using recessed lighting in the ceiling, formulas can be used to determine the number of light fixtures that should be used and their placement in the room.

The following formula can be solved to determine the maximum number of light fixtures (m) to use in the kitchen you are decorating:

$$
5(3 m+2)+12 \geq 20 m-18
$$

You will have the maximum number of light fixtures ( $\mathbf{m}$ ) installed at a cost of $\$ 65$ each. This price already includes the installation by an electrician.

## Part 5 - Roofing Shingles

New roofing shingles will be installed. The roof is made up of two identical rectangles, one of which you can see in grey in the diagram below (ABCD).

The cost for the shingles and the installation can be calculated by using the following function
 where $\mathbf{C}(\mathbf{a})$ is the cost and $\mathbf{a}$ is the area of the roof to be covered, in $\mathrm{m}^{2}$ :

$$
C(a)=11.50 a
$$

## Part 1 - The Painters

Company A: $\mathrm{C}(\mathrm{h})=\mathbf{2 4 . 5 h}+130$
Company B: C(h) = 26.5h+96
Solve the system of equations:

| $\left.\begin{array}{c}24.5 h+130=26.5 h+96 \\ -24.5 h \\ \hline 130\end{array}\right)=2 h+96$ |
| :---: |
| $-96 \quad-96$ |
| $\frac{34}{2}=\frac{2 h}{2}$ |
| $\underline{17}=$ |

Calculate the cost:
Company A: $\quad C(h)=24.5 h+130$

$$
C(17)=24.5(17)+130=546.50
$$

Company B: $\quad C(h)=26.5 h+96$

$$
C(17)=26.5 h+96=546.50
$$

So, for 17 hours of work, both companies charge $\underline{\$ 546.50}$.

## Part 2 - Carpets

Area of bedroom 1
$A=l \bullet w$
$A=x \bullet(x+2)$
$\underline{A}=x^{2}+2 \underline{x}$

Area of bedroom 2

$$
\begin{aligned}
& A=l \bullet w \\
& A=(x+1) \bullet(x+0.8) \\
& A=x^{2}+0.8 x+x+0.8 \\
& \underline{A}=x^{2}+1.8 x+0.8
\end{aligned}
$$

Area of bedroom $1=$ Area of bedroom 2

$$
\begin{gathered}
x^{2}+2 x=x^{2}+1.8 x+0.8 \\
\frac{-x^{2}}{2 x=1.8 x+0.8} \\
\frac{-1.8 x-1.8 x}{0.2 x}=\frac{0.8}{0.2} \\
\frac{0.2}{0.2}=\frac{x=4}{2}
\end{gathered}
$$

Therefore, the dimensions of bedroom 1 are: $\quad x=4 \mathrm{~m}$ and

$$
x+2=4+2=6 m
$$

the dimensions of bedroom 2 are:

$$
x+1=4+1=\overline{5 m} \text { and }
$$

$$
x+0.8=4+0.8=\underline{\underline{4.8} \mathrm{~m}}
$$

Area of bedroom 1
Area of bedroom 2
$\underline{A}=l \bullet w=6 \bullet 4=24 m^{2}$

$$
A=l \bullet w=5 \bullet 4.8=24 m^{2}
$$

Total area for both bedrooms $=\mathbf{2 4} \boldsymbol{m}^{2}+24 \boldsymbol{m}^{2}=48 \boldsymbol{m}^{2}$

Cost for the carpets $=$ area $\bullet \$ 55 / m^{2}=48 \bullet 55=\underline{\underline{\$ 2640.00}}$

## Part 3 - Curtains

Let: $\boldsymbol{n}$ be the number of windows
$\boldsymbol{C}(\boldsymbol{n})$ be the cost
Cost to purchase the curtains:

$$
\begin{aligned}
& a=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{1579.50-877.50}{9-5}=\frac{702}{4}=\$ \mathbf{1 7 5 . 5 0} / \text { window } \\
& b=y-a x=877.50-(175.50)(5)=\mathbf{0}
\end{aligned}
$$

Therefore, the rule for the curtains:

$$
\begin{aligned}
& y=a x \\
& \underline{C}(n)=175.50 n
\end{aligned}
$$

The cost for the curtains for 12 windows: $C(12)=175.50 \bullet 12=\underline{\$ 2106.00}$

## Part 4 - Recessed Lighting

Maximum number of light fixtures:
$5(3 m+2)+12 \geq 20 m-18$
$15 m+10+12 \geq 20 m-18$
$15 m+22 \geq 20 m-18$
$-22-22$
$15 m \geq 20 m-40$
$-20 m-20 m$
$-5 m \geq-40$
$\frac{-5 m}{-5} \leq \frac{-40}{-5}$
$m \leq 8$

Cost for the light fixtures = \$65/light $\bullet$ number of lights $=65 \bullet 8=\underline{\mathbf{\$ 5 2 0 . 0 0}}$

## Part 5 - Roofing Shingles

Length of hypotenuse $\overline{A D}$ :
Measure of side $\overline{D E}=10 \mathrm{~m} / 2=5 \mathbf{m}$
$c=\sqrt{a^{2}+b^{2}}$
$c=\sqrt{5^{2}+3.5^{2}}$
$c=\sqrt{25+12.25}$
$c=\sqrt{37.25}$
$\underline{c} \approx 6.10 \mathrm{~m}$
Area of rectangle $A B C D=$ length $x$ width $\approx 14 \times 6.10 \approx \underline{\underline{85.46} \mathrm{~m}^{2}}$
Area of both rectangles $\approx 85.46 \mathrm{~m}^{2} \times 2 \approx 170.89 \mathrm{~m}^{2}$
Cost for the roofing shingles:
$C(a)=11.50 a$
$C(170.89) \approx 11.50 \bullet 170.89 \approx \$ 1965.26$

## Summary of expenses:

| Part | Cost (\$) |
| :---: | :---: |
| The Painters | 546.50 |
| Carpets | 2640.00 |
| Curtains | 2106.00 |
| Recessed Lighting | 520 |
| Roofing Shingles | 1965.26 |
| Total | $\mathbf{7 7 7 7 . 7 6}$ |

The house renovations will cost $\$ 7777.76$.
Is the cost within budget? YES, there will be $\mathbf{\$ 2 2 2 . 2 4}$ left in the budget.
NO, there will be a deficit of $\$$ $\qquad$ .

